Corona Calculator

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1. HowTo CoronaCalculator.xlsm

1. Download “CoronaCalculator.zip”.

2. Unzip “CoronaCalculator.zip”. It contains “CoronaCalculator.xlsm”.

3. Open “CoronaCalculator.xlsm” with MS EXCEL.

4. Look at the Menu Bar and locate the entry “Corona”.

5. Choose this entry. Now you see 2 buttons:

   “Drop Charts”
   “Corona Prognosis”

6. Choose “Corona Prognosis”. Now you see a dialog. It contains 5 textboxes with labels:

   “Incubation Period (in days)”

   “r-Value Before (per day)”
   “r-Value Target (per day)”

   “lost (in days)”

   “Infected Humans (at zero)”

7. The textbox with label “Incubation Period (in days)” allows integers >= 1. It contains the time for incubation.

8. The textbox with label “r-Value Before (per day)” allows floats > 0. It contains the r-Value before the new Corona-Strategy.

9. The textbox with label “r-Value Target (per day)” allows floats > 0. It contains the intended r-Value for the new Corona-Strategy.

10. The textbox with label “lost (in days)” allows integers >= 0. It contains the lost time before the new Corona-Strategy is applied.

11. The textbox with label “Infected Humans (at zero)” allows integers >= 0. It contains the number of infected humans at time zero.

12. Enter the values and press “Plot”.

2. Data Description

The “CoronaCalculator.xlsm” contains 5 sheets:

1. The sheet “r-Values” contains the transition a *r*-Value to another *r*-Value.

2. The sheet “Infect Rate” contains the evolution of the infect rate for a year. It is caused by the evolution of the r-Value.

3. The sheet “Detail lost” is the detailed view of the infect rate for (2 * lost) days

4. The sheet “Detail Effect” is the detailed view of the infect rate for (2 * (lost + Incubation Period)) days.

5. The sheet “Infections” contains the evolution of infected humans for a year. There you can see the total effect of the new Corona-Strategy.

3. The IVP

Let I be an interval of $\mathbb{R}$ with $I \neq \varnothing$.
Let $\xi \in I$.
Let $\eta \in \mathbb{R}$.
The IVP, that describes infections, is well known. It is:

$$y'(x) = \alpha(x)\, y(x)$$

$$y(\xi) = \eta$$

with a given continuous function $\alpha : I \to \mathbb{R}$

It is a linear ODE. The solution of this IVP is also well known and can be found in "Walter, Gewöhnliche Differentialgleichungen, Springer-Verlag, ISBN 3-540-16143-0".

4. The Simplest IVP

The most simplest approach to a concret infection is the assumption:

$\alpha(x)$ is constant with a value $c \in \mathbb{R}$

Then the IVP has a very simple solution. It is:

$$y(x) = \eta \cdot e^{c \cdot (x - \xi)}$$

In the case of $\eta \neq 0$ we can define the so-called r-Value of the IVP. It is:

$$r := \frac{y(1)}{y(0)} = \frac{e^{c \cdot (1 - \xi)}}{e^{c \cdot (-\xi)}} = e^{c} > 0$$

i.e.

$$c = \ln(r)$$

If two different IVPs of the simplest kind have the same r-Value, then the xy-Charts of the solutions of these IVPs only differ in the scalings of the Axes.

For example, the simplest IVP describes the evolution of a bacteria culture quite well.

Cave!: There are physically limits for this model:

1. Someday earth is consumed.

2. Someday the spatial spreading of cell division exceeds the speed of light.

5. A More Complicated IVP (V1)

Let $\xi, \eta \in \mathbb{R}$.
Let $\gamma, \delta \in \mathbb{R}$ with $\delta \neq 0$.
We define the function $\alpha : \mathbb{R} \to \mathbb{R}$ through

$$\alpha(x) = \gamma + \delta \cdot (x - \xi)$$

We the define another function $\beta : \mathbb{R} \to \mathbb{R}$ through

$$\beta(x) = \gamma \cdot (x - \xi) + \frac{1}{2} \cdot \delta \cdot (x - \xi)^2$$

Then we have:

$\alpha : \mathbb{R} \to \mathbb{R}$ and $\beta : \mathbb{R} \to \mathbb{R}$ are C^∞
$\beta : \mathbb{R} \to \mathbb{R}$ is an antiderivative of $\alpha : \mathbb{R} \to \mathbb{R}$
$\beta(\xi) = 0$

The IVP

$$y'(x) = \alpha(x)\, y(x)$$
$$y(\xi) = \eta$$

has the solution:

$$y(x) = \eta \cdot e^{\beta(x)}$$

6. A More Complicated IVP (V2)

Let $\gamma \in \mathbb{R}$ with $\gamma > 0$.
Let $\delta \in \mathbb{R}$ with $\delta \neq 0$.
Let $\xi \in \mathbb{R}$.
Let $\eta \in \mathbb{R}$.
We define an interval $I := \left]\xi - \frac{\gamma}{|\delta|} ; \xi + \frac{\gamma}{|\delta|}\right[$.

We define the function $\alpha : I \to \mathbb{R}$ through

$$\alpha(x) = \ln\left(\gamma + \delta \cdot (x - \xi)\right)$$

With "Bronstein, Semendjajew, Taschenbuch der Mathematik, Verlag Harri Deutsch, ISBN 3-871-44492-8" we have:

$$\begin{aligned}
\alpha(x) &= \ln\left(\gamma \cdot \left(1 + \frac{\delta}{\gamma} \cdot (x - \xi)\right)\right) = \\
&= \ln(\gamma) + \ln\left(1 + \frac{\delta}{\gamma} \cdot (x - \xi)\right) = \\
&= \ln(\gamma) + \sum_{i=1}^{\infty} (-1)^{i+1} \cdot \frac{\left(\frac{\delta}{\gamma} \cdot (x - \xi)\right)^i}{i} = \\
&= \ln(\gamma) + \sum_{i=1}^{\infty} \frac{(-1)^{i+1} \cdot \left(\frac{\delta}{\gamma}\right)^i}{i} \cdot (x - \xi)^i
\end{aligned}$$

We can now define another function $\beta : I \to \mathbb{R}$ through

$$\beta(x) = \ln(\gamma) \cdot (x - \xi) + \sum_{i=1}^{\infty} \frac{(-1)^{i+1} \cdot \left(\frac{\delta}{\gamma}\right)^{i}}{i \cdot (i + 1)} \cdot (x - \xi)^{i+1}$$

Then we have obviously:

$\alpha : I \to \mathbb{R}$ and $\beta : I \to \mathbb{R}$ are C^{∞}
$\beta : I \to \mathbb{R}$ is an antiderivative of $\alpha : I \to \mathbb{R}$
$\beta(\xi) = 0$

The IVP

$$y'(x) = \alpha(x)\, y(x)$$
$$y(\xi) = \eta$$

has the solution:

$$y(x) = \eta \cdot e^{\beta(x)}$$

7. A Real IVP (V1): Transition Of *r*-Values

Let $r_1, r_2 \in \mathbb{R}_+$.
Let $x_1, x_2 \in \mathbb{R}$ with $0 \le x_1 < x_2$.
We define $c_1, c_2 \in \mathbb{R}$ through

$$c_1 := \ln\left(r_1\right)$$
$$c_2 := \ln\left(r_2\right)$$

We define the function $\alpha : \mathbb{R} \to \mathbb{R}$ through

$$\alpha(x) := \begin{cases} c_1 & x \le x_1 \\ c_1 + \dfrac{x - x_1}{x_2 - x_1} \cdot \left(c_2 - c_1\right) & x_1 \le x \le x_2 \\ c_2 & x_2 \le x \end{cases}$$

Knowing 4. and 5. we can solve the associated IVP.

8. A Real IVP (V2): Transition Of *r*-Values

Let $r_1, r_2 \in \mathbb{R}_+$ with $r_1 \neq r_2$ and $r_2 < 2 \cdot r_1$.
Let $x_1, x_2 \in \mathbb{R}$ with $0 \leq x_1 < x_2$.
We define $c_1, c_2 \in \mathbb{R}$ through

$$c_1 := \ln\left(r_1\right)$$
$$c_2 := \ln\left(r_2\right)$$

We define the function $\alpha : \mathbb{R} \to \mathbb{R}$ through

$$\alpha(x) := \begin{cases} c_1 & x \leq x_1 \\ \ln\left(r_1 + \dfrac{x - x_1}{x_2 - x_1} \cdot \left(r_2 - r_1\right)\right) & x_1 \leq x \leq x_2 \\ c_2 & x_2 \leq x \end{cases}$$

We define $\gamma \in \mathbb{R}_+$ and $\delta \in \mathbb{R}$ with $\delta \neq 0$ through

$$\gamma := r_1 > 0$$
$$\delta := \frac{r_2 - r_1}{x_2 - x_1} \neq 0$$

We examine the function $\alpha : \mathbb{R} \to \mathbb{R}$. We set $\xi \in \mathbb{R}$ and $I \subseteq \mathbb{R}$ as

$$\xi := x_1$$
$$I := \left]\xi - \frac{\gamma}{|\delta|} ; \xi + \frac{\gamma}{|\delta|}\right[$$

Then we have:

$$x_1 = \xi \in I$$

$$\delta = \frac{r_2 - r_1}{x_2 - \xi} \neq 0$$

$$\alpha(x) = \begin{cases} c_1 & x \leq \xi \\ \ln\left(\gamma + \delta \cdot (x - \xi)\right) & \xi \leq x \leq x_2 \\ c_2 & x_2 \leq x \end{cases}$$

We want to apply 6. So we must show:

$$x_2 \in I$$

Proof of that:

$$x_2 \in I \quad \Leftrightarrow$$

$$\xi - \frac{\gamma}{|\delta|} < x_2 < \xi + \frac{\gamma}{|\delta|} \quad \Leftrightarrow$$

$$-\frac{\gamma}{|\delta|} < x_2 - \xi < \frac{\gamma}{|\delta|} \quad \Leftrightarrow$$

$$-r_1 \cdot \frac{x_2 - \xi}{\left|r_2 - r_1\right|} < x_2 - \xi < r_1 \cdot \frac{x_2 - \xi}{\left|r_2 - r_1\right|}$$

Because of $x_2 > x_1 = \xi$ and $r_1 > 0$, we get:

$$
\begin{aligned}
& x_2 \in I \quad \Leftrightarrow \\
& x_2 - \xi < r_1 \cdot \frac{x_2 - \xi}{\left|r_2 - r_1\right|} \quad \Leftrightarrow \\
& 1 < r_1 \cdot \frac{1}{\left|r_2 - r_1\right|} \quad \Leftrightarrow \\
& \left|r_2 - r_1\right| < r_1 \quad \Leftrightarrow \\
& -r_1 < r_2 - r_1 < r_1 \quad \Leftrightarrow \\
& 0 < r_2 < 2 \cdot r_1
\end{aligned}
$$

Because of $r_1, r_2 \in \mathbb{R}_+$, we get:

$$x_2 \in I \quad \Leftrightarrow \quad r_2 < 2 \cdot r_1$$

Because I is an interval of $\mathbb{R}$ and $x_1, x_2 \in I$, it follows:

$$\left[x_1; x_2\right] \subseteq I$$

Knowing 4. and 6. we can solve the associated IVP.

9. More About The *r*-Value

9.1. Interpretation

The *r*-Value compares per definitionem the states of the infect between the times 1 and 0.

For example:

A value of 1.14 for the *r*-Value of the IVP means that 100 humans in average infect 114 another humans in the time from 0 until 1. The *r*-Value is dependent from the time unit you use.

9.2. Conversion of the *r*-Value from "per day" to "per week"

We have:

$$r_W := \left(r_D\right)^7$$

9.3. Conversion of the *r*-Value from "per week" to "per day"

We have:

$$r_D := \sqrt[7]{r_W} = \left(r_w\right)^{\frac{1}{7}}$$

9.4. Remark

There are several definitions of the *r*-Value out there. The *r*-Value, which is used here, originates from my education.

If you compute an average *r*-Value, you must use the geometric average.

10. Incubation Period

There are sevaral possibilities for the Incubation Period. Please experiments with it:

1. The average Incubation Period is - as far as I googled – circa 7 days.

   So you can try 7 days.

2. The average Incubation Period is the time, when half the infections are broken out.

   So you can try 14 = 2 * 7 days.

3. The maximal Incubation Period is – as far as I googled – circa 12 Day.

   So you can try 12 days.

11. Total Infections

To compute the number of total infections from time 0, you must integrate the solution of the real IVP.

In the case of $x \leq x_1$ or $x_2 \leq x$ you can find an antiderivative in “Bronstein, Semendjajew, Taschenbuch der Mathematik, Verlag Harri Deutsch, ISBN 3-871-44492-8”

In the case $x_1 \leq x \leq x_2$ you must compute an antiderivative numerically. The algorithm can be found in “Stoer, Bulirsch, Numerische Mathematik 2, Springer-Lehrbuch, ISBN 3-540-51482-1”. It is called “Euler’s Polygonal Method”.