# A Bijection $\mathbb{N}_0 \rightarrow (\mathbb{N}_0)^2$

Let $q \in \mathbb{N}$ be with $q \geq 2$.
A set $A_q$ is defined as

$$A_q := \left\{ a \in \{0, \ldots, q-1\}^{(\mathbb{N}_0)} : \exists k \in \mathbb{N}_0 \ \forall_{\substack{j \in \mathbb{N}_0 \\ j \geq k}} \ a_j = 0 \right\}$$

Then there is one and only one mapping $\Phi_{2,q} : \mathbb{N}_0 \rightarrow (\mathbb{N}_0)^2$ with

$$\forall a \in A_q \quad \Phi_{2,q}\left( \sum_{j=0}^{\infty} a_j \ q^j \right) = \begin{pmatrix} \sum_{j=0}^{\infty} a_{2j} \ q^j \\ \sum_{j=0}^{\infty} a_{2j+1} \ q^j \end{pmatrix}$$

Obviously the following is true:

$\Phi_{2,q} : \mathbb{N}_0 \rightarrow (\mathbb{N}_0)^2$ is a bijection

Remark:

The above infinite sums are de facto only finite sums.

Example:

$$\Phi_{2,10}(204040152364) = \begin{pmatrix} 534 \\ 244126 \end{pmatrix}$$