

1. Boolean Algebra

We are trying here to define the Boolean Algebra with its operations. We shall see later, why this attempt must fail.

Let be given two different objects T and F .

We define an operation $\neg: \{T, F\} \rightarrow \{T, F\}$ through

	T	F
$\neg \dots$	F	T

i.e. with other words

$$\forall X \in \{T, F\} \quad \neg X \text{ means "not" } X$$

We define an operation $\wedge: \{T,F\} \times \{T,F\} \rightarrow \{T,F\}$ through

$\dots \wedge \dots$	T	F
T	T	F
F	F	F

i.e. with other words

$\forall X, Y \in \{T, F\}$ $X \wedge Y$ means "X 'and' Y"

We define an operation $\vee: \{T,F\} \times \{T,F\} \rightarrow \{T,F\}$ through

$\dots \vee \dots$	T	F
T	T	T
F	T	F

i.e. with other words

$\forall X, Y \in \{T, F\}$ $X \vee Y$ means "X 'or' Y"

2. The Alien-Point-of-View

An alien might try the following to define his Boolean Algebra:

Let be given two different objects \circ and \bullet .

Definition \blacktriangle : $\{\circ, \bullet\} \rightarrow \{\circ, \bullet\}$:

	\circ	\bullet
$\blacktriangle\dots$	\bullet	\circ

Definition \blacksquare : $\{\circ, \bullet\} \times \{\circ, \bullet\} \rightarrow \{\circ, \bullet\}$:

$\dots\blacksquare\dots$	\circ	\bullet
\circ	\circ	\bullet
\bullet	\bullet	\bullet

Definition \square : $\{\circ, \bullet\} \times \{\circ, \bullet\} \rightarrow \{\circ, \bullet\}$:

$\dots\square\dots$	\circ	\bullet
\circ	\circ	\circ
\bullet	\circ	\bullet

3. Where Is The Problem?

We assume that we succeeded to define the so called Boolean Algebra $(T, F, \neg, \wedge, \vee)$ in section 1.

The problem is:

You cannot decide, wether $(O, \bullet, \blacktriangle, \blacksquare, \square)$ in section 2. means $(T, F, \neg, \wedge, \vee)$ or $(F, T, \neg, \vee, \wedge)$. The attempt to define $(O, \bullet, \blacktriangle, \blacksquare, \square)$ fails, because it does not distinguish between the two interpretations $(T, F, \neg, \wedge, \vee)$ and $(F, T, \neg, \vee, \wedge)$ (Cave $T \neq F!$).

The attempt in section 1. fails also. The reason, that it seems to be pausable, but it is not, is, that it is colloquial true, what is meant by 'true', 'false', 'not', 'and' and 'or', but these terms cannot be defined to full satisfaction.

By the way:

It does not help to give the objects in section 2 names.

4. Literature

- [1] Lectures in Mathematics 1987 - 2002
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