Sums

Theorem:

Pre.: Let $n \in \mathbb{N}_+$ with $n \geq 2$.

Ass.: The following ist true:

$$\sum_{i=1}^{n-1} i = \sum_{i=1}^{n-1} (n - i) = \frac{1}{2} n (n - 1)$$

and

$$\sum_{i=1}^{n} i = \frac{1}{2} n (n + 1)$$

Proof: These formulas are known.

Combination of Sums

Theorem:

Pre.: Let $n \in \mathbb{N}_+$ with $n \geq 2$.

Ass.: The following is true:

$$\left(\sum_{i=1}^{n-1} i \right) + n + \left(\sum_{i=1}^{n-1} (n - i) \right) = n^2$$

Proof: The proof is trivial.